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Abstract

This paper addresses the transportation problem using public electric cars. At each car station, a decision, concerning the cars which should become available to customers, has to be taken. We assume that a vehicle is available when its charge is greater than a given threshold. Our goal is to optimize this threshold.

Keywords: Logistics, Electric Cars, Self Service Cars ...

Véhicules électriques : Effet du seuil de disponibilité sur le coût de transport.

Résumé

Cette communication s'intéresse au problème de transport basé sur des véhicules électriques individuels. Il s'agit de décider, en fonction de leur charge électrique, des véhicules qui seront disponibles pour les utilisateurs. Nous supposons qu'un véhicule est disponible lorsque sa charge dépasse un certain seuil. Nous cherchons à définir ce seuil.

Mots-clés : Logistique, Véhicules électriques, Véhicules en libre service...

1 Introduction

The system, studied in this paper, consists of a pool of electric cars spread out over a set of stations. A customer picks up an electric car at one station, uses it for a short period of time and returns it to another (or the same) station. In other words, a car is used by different customers each day, hence the interest of the system. Our aim is to obtain a high satisfaction rate, by providing cars to the largest number of customers as soon as they arrive at a station.

Why are we using electric cars?

1. To reduce pollution in the city. Actually an electric car is silent and non-polluting, no emissions will harm the environment.
2. Electric cars have a good city performance characteristics. The peak speed lies between 60 and 80 km/h, and finally these cars are peppy so they can be fun to drive.
3. To reduce cost. Electric cars are more economical than fuel cars.

What are the limitations of an electric car?

1. An electric car, fully charged, can cover a distance of 100 km.
2. The recharge duration time of a car is eight hours.
3. From time to time, the batteries need to be completely emptied to readjust the gauge.

Clients' requirements

1. Clients do not like to wait (at most, five minutes). As soon as they arrive at a car station they want a car ready for a journey.
2. They want a well-kept car.
3. During a journey, they don't accept to have a breakdown. It can be due to several reasons such as.
   a. Technical reasons (either mechanical or electrical).
   b. Running out of energy. To avoid this problem, a car is available for clients only if its level of energy is greater than a determined threshold that guarantees the well accomplishment of a journey.

It is obvious that at some times of the day, we will have either an overflow (due to the capacities of the sites) or a shortage of cars (an underflow) at one or more stations. An optimized system which redistributes the

available cars over the set of stations must be developed. In this paper we assume that this system of balancing is perfect. In other words, we assume that when a car leaves a station, it is automatically replaced by another one available for clients. It's a way to separate the balancing problem from the one of defining the optimal threshold. Consequently, the ability to manage the system optimaly is conditional on the optimal threshold.
As we said before, a car is available if the charge level is greater than a certain threshold. Our aim in this paper is to determine this threshold.

2 Problem Formulation

2.1 Objective Function

Our criterion for a given threshold is the clients' dissatisfaction rate. This rate O(s) is evaluated as follows:

$$O(s) \;=\; c_U \; E[U(s)] + c_R \; E[R(s)] \tag{1}$$

where:

- s is the charge level threshold that determines whether a car is available for a client or not.
- E[U(s)] is the expectation of the number of unsatisfied clients in a station. This is for one of two reasons:
  1. A shortage of cars in the station. This case will not be considered in this paper as we assumed that the balancing process is perfectly implemented.
  2. The car is physically here but has not enough energy to complete a journey.

  Thus, E[U(s)], concerns only the second dissatisfaction reason.
- E[R(s)] is the expectation of the number of cars which will run out of energy during a trip. In this paper, we will call it, expectation of running out of energy. This expectation, as we will see later, depends on the threshold s .
- c_U is the cost when a client asks for a car which is unavailable. This cost includes the waiting time of a client, and his dissatisfaction.
- c_R is the cost when a car runs out of energy during a trip. When a client has a breakdown during a journey, the car must be towed as far as a station, moreover, the system must compensate the client by paying him a cab.

Hereafter, we explain how we calculated this criterion.

2.2 Calculation of E[U(s)], Expectation of Unsatisfied Clients in a Car Station.

We assume that a car is available for clients if its charge level is greater than a certain threshold **s** . The charge level **y** for an available car lies between [s, C], where **C** is the maximal car charge level.
At a car station, we have either available or unavailable cars. We assume that the charge level of a car is uniformly distributed between 0 and C. Thus, the proportion of available cars in a car station is

$$B(A) = \frac{C - s}{C}$$

while for the unavailable cars it is expressed as:

$$B(U) = \frac{s}{C}$$

Let N be the random variable representing the number of cars in a car station.
Hence, the probability for the random variable A representing the number of available cars in a car station is determined by a binomial distribution.

$$\begin{aligned} P(A = a/N = n) &= C_n^a \; (B(A))^a \; (1 - B(A))^{n-a} \\ &= C_n^a \; (\tfrac{C-s}{C})^a \; (\tfrac{s}{C})^{n-a} \end{aligned}$$

Notice :

1. $0 \leq a \leq n$ where **n** is the total number of cars (availables and unavailables) in the car station considered.
2. $C_n^a = \frac{n!}{a!(n-a)!}$

The expectation E[U(s)] of unsatisfied clients in a car station, as we said before, is the expectation E[U(s)] of the number of clients unsatisfied due to the immobilization of some cars. Remember the starting hypothesis which is 'the balancing process is perfect'. Therefore, the number of demands is less than or equal to the number of cars in the station $k \leq n$.

Let $h_N(n)$ be the probability density of **N** .

$$E[U(s)] = \sum_{n} \sum_{k=0}^{n} \sum_{a=0}^{k} P(K = k)\ P(A = a/N = n)\ (k - a)\ h_N(n)$$

let w = k - a.

$$E[U(s)] = \sum_{n} \sum_{k=0}^{n} \sum_{w=0}^{k} P(K = k)\ P(A = k - w/N = n)\ w\ h_N(n)$$

where:

- **K** is the random variable representing the number of clients' demands in the car station. We assume that the probability of having **k** demands is known.
- **A** is the random variable representing the number of available cars in the car station.
- **w=k-a** is the number of unsatisfied demands due to the fact that some cars are underloaded.

Finally, the expectation of unsatisfied clients in a car station E[U(s)] is:

$$E[U(s)] = \sum_{n} \sum_{k=0}^{n} \sum_{w=0}^{k} P(K = k)\ C_n^{k-w} (\frac{C - s}{C})^{k-w} (\frac{s}{C})^{(n-k+w)}\ w\ h_N(n) \tag{2}$$

where: $C_n^{k-w} = \frac{n!}{(k-w)!(n-(k-w))!}$

2.3 Calculation of E[R(s)], Expectation of Running Out of Energy.

A car whose maximal level of charge is **C** can cover a distance L. We assume that a trip will never reach the distance **L** but will be limited to distances between [0, G] such that $G \leq L$.

In other words, in order to cover the maximal distance **G** , a car needs a charge level **y** which is equal to $\frac{CG}{L}$, and to cover a distance x, the charge level **y** is equal to:

$$y = \frac{C}{L}x \tag{3}$$

A car will be ready for use, if its charge level is greater than the threshold s . Such a car, in this paper, is called an available car. As we said before, a car is available if its charge level **y** lies between [s, C], i.e.:

$$s \leq y \leq C \tag{4}$$

A car runs out of energy during a trip, if the trip requires a charge level **y** greater than the car charge level. In other words when

$$y < \frac{C}{L}x \quad or \quad x > \frac{L}{C}y$$

where:

- **y** is the car charge level.
- **x** is the distance of the trip.
- $\frac{C}{L}x$ is the charge level needed to cover a trip of distance **x** .
- $\frac{L}{C}y$ is the distance that the car can cover with a charge level **y** .

The probability that a car of charge level **y** will run out of energy during a trip, is equal to the probability that the distance of the trip **x** is greater than the distance that the car is able to cover, knowing its charge level. This is calculated as follows:

$$P(X > \frac{L}{C}y/Y = y) = \int_{\frac{L}{C}y}^{\infty} g_X(x)dx \tag{5}$$

where:

- $\mathbf{X}$ is the random variable representing the distance of a trip.
- $\mathbf{Y}$ is the random variable representing the available car charge level.
- $\frac{L}{C}y$ is the distance that the car can cover with a charge level $\mathbf{y}$.
- $g_X(x)$ is the probability density of the random variable $\mathbf{X}$.

$q_R(s)$ is the probability that a car will run out of energy during a trip. To calculate $q_R(s)$, we consider each possible charge level $\mathbf{y}$ (for an available car) individually, and compute the probability that the car will run out of energy knowing its charge level y, multiplying it by the probability density of this charge level. We can express it as follows.

$$q_R(s) = \int_s^C (P(X > \frac{yL}{C}/Y = y))\ f_Y(y)\ dy \tag{6}$$

where:

- $f_Y(y)$ is the probability density of the random variable $\mathbf{Y}$ representing the charge level of an available car.
- $P\ (X > \frac{yL}{C}/Y = y)$ is the probability that a car of charge level $\mathbf{y}$ will run out of energy during a journey determined in (5).

Remark: The limits of Integration are s and C, because the charge level $\mathbf{y}$ for an available car lies between [s, C], (see 4).

The expectation E[R(s)] of running out of energy, as we said before, is the expectation of the number of cars which will run out of energy during a trip. We formulate this as follows:

$$E[R(s)] = \sum_n \sum_{d=0}^{n} \sum_{l=0}^{d} P(D = d)\ P(L = l/D = d)\ l\ h_N(n) \tag{7}$$

where:

- D is the random variable representing the number of available cars that leave the car station.
- $\mathbf{d}$ is the number of cars that leave the car station.

- L is the random variable representing the number of cars which will run out of energy during the trip.
- l is the number of cars that run out of energy during a trip.
- $l \leq d$. It is obvious that the number of cars which ran out of energy during a trip is less than or equal to the number of cars that effectively left the station.
- $P(L = l/D = d)$ is the probability to have l out of d cars whose energy is not enough to complete their trips.
- $P(D = d)$ is the probability that d available cars (satisfied clients) leave the car station.
- $h_N(n)$ is the random variable representing the number of cars in a car station.

When a car leaves a car station, either it will complete the journey successfully or it will run out of energy during the trip and will not cover the whole distance. By a binomial distribution, we can determine $P(L = l/D = d)$, the probability that l out of d cars will run out of energy during a trip as:

$$P(L = l/D = d) = C_d^l \ (q_R(s))^l \ (1 - q_R(s))^{d-l}$$

where:

- $q_R(s)$ is the probability that a car will run out of energy during a trip, calculated in (6).
- $C_d^l = \frac{d!}{l!(d-l)!}$.

If we replace $P(L = l/D = d)$ by its value in equation (7), we obtain:

$$E[R(s)] = \sum_{n} \sum_{d=0}^{n} \sum_{l=o}^{d} P(D = d) \ C_d^l (q_R(s)^l)(1 - q_R(s))^{d-l} l \ h_N(n)$$

but, according to the expectation of a binomial distribution

$\sum_{l=0}^{d} C_d^l \ (q_R(s))^l \ (1 - q_R(s))^{d-l} \ l \ = \ d \ q_R(s)$

then: $E[R(s)] \; = \; \sum_n \; \sum_{d=0}^{n} \; d \; q_R(s) \; P(D=d) \; h_N(n)$

or $\quad E[R(s)] \; = \; q_R(s) \; \sum_n \; \sum_{d=0}^{n} \; d \; P(D=d) \; h_N(n)$

where $\sum_n \; \sum_{d=0}^{n} \; d \; P(D=d) \; h_N(n) \; = \; E[D(s)]$ which is the expectation of the number of available cars that leave the station.

Finally we obtain:

$$E[R(s)] = q_R(s) \; E[D(s)] \tag{8}$$

We assumed that the total number of cars in the car station covers all the clients' demands **k**. From these k demands, some are satisfied while the others are not, because the cars are immobilized due to the threshold policy. So we can express the expectation E[D(s)] of the number of cars that leave the car station as follows.

$$E[D(s)] = E[K] - E[U(s)] \tag{9}$$

where:

- $E[U(s)]$ is the expectation of unsatisfied clients in a car station that we have already calculated in (2).
- $E[K]$ is the expectation of the number of clients' demands and is equal to $\sum_k kP(K=k)$

The previous equation (9) is demonstrated as follows:

Two cases have to be considered to define the number of cars which leave the car station.

1. All clients' demands are satisfied because the number of available cars is enough to cover all the demands.
2. All clients' demands are not satisfied because the number of available cars isn't enough to cover all the demands. Only available cars leave the car station.

$$E[D(s)] \;=\; \sum_n \sum_{d=0}^{n} d\ P(K=d) \sum_{a=0\ a>d}^{n} P(A=a/N=n)\ h_N(n)$$

$$+\ \sum_n \sum_{a=0}^{n} a\ P(A=a/N=n)\ \sum_{d=0\ a\le d}^{n} P(K=d)\ h_N(n)$$

$$E[D(s)] = \sum_n \sum_{d=0}^{n} d\ P(K=d)$$

$$* \left(1 - \sum_{a=0\ a\le d}^{n} P(A=a/N=n)\right)\ h_N(n)$$

$$+ \sum_n \sum_{a=0}^{n}\ a\ P(A=a/N=n)\ \sum_{d=0\ a\le d}^{n} P(K=d)\ h_N(n)$$

$$E[D(s)] = \sum_n \sum_{d=0}^{n} d\ P(K=d)\ h_N(n)$$

$$- \sum_n \sum_{d=0}^{n} d\ P(K=d)\ \sum_{a=0\ a\le d}^{n} P(A=a/N=n)\ h_N(n)$$

$$+ \sum_n \sum_{a=0}^{n} a\ P(A=a/(N=n)\ \sum_{d=0\ a\le d}^{n} P(K=d)\ h_N(n)$$

$$E[D(s)] = \sum_n \sum_{d=0}^{n} d\ P(K=d)\ h_N(n)$$

$$- \sum_n \sum_{d=0}^{n} \sum_{a=0\ a\le d}^{n} d\ P(K=d)\ P(A=a/N=n)\ h_N(n)$$

$$+ \sum_n \sum_{d=0}^{n} \sum_{a=0\ a\le d}^{n} a\ P(A=a/N=n)\ P(K=d)\ h_N(n)$$

$$E[D(s)] = \sum_n \sum_{d=0}^{n} d\ P(K=d)\ h_N(n)$$

$$- \sum_n \sum_{d=0}^{n} \sum_{a=0\ a\le d}^{n} P(A=a/N=n)$$

$$* P(K=d)\ (d-a)\ h_N(n)$$

Let w = d-a

$$E[D(s)] = \sum_n \sum_{d=0}^{n} d\ P(K=d)\ h_N(n)$$

$$- \sum_n \sum_{d=0}^{n} \sum_{w=0}^{d} P(A=d-w/N=n)\ P(K=d)\ w\ h_N(n)$$

then: $E[D(s)] = E[K]\ -\ E[U(s)]$

Finally, $E[R(s)]$, the expectation of running out of energy, is equal to:

$$E[R(s)] = q_R(s)\ E[D(s)]$$
$$= q_R(s)\ (E[K] - E[U(s)])$$
$$E[R(s)] = q_R(s)\ E[K] - q_R(s)\ E[U(s)] \tag{10}$$

When replacing each term by its value, we obtain :

$$E[R(s)] = \int_s^C (\int_{\frac{yL}{C}}^{\infty} g_X(x)\ dx)\ f_Y(y)\ dy\ [\ \sum_k kP(K = k)$$
$$- \sum_n \sum_{k=0}^n \sum_{w=0}^k P(K = k)\ P(A = k - w/N = n)\ w\ h_N(n)\ ]$$

2.4 Formulation of the Objective Function

$$O(s) = c_U\ E[U(s)] + c_R\ E[R(s)]$$

Remember equations (10) and (2) :

$$E[U(s)] = \sum_n \sum_{k=0}^n \sum_{w=0}^k P(K = k)\ P(A = k - w/N = n)\ w\ h_N(n)$$
$$E[R(s)] = q_R(s)\ E[K] - q_R(s)\ E[U(s)]$$

knowing that $E[K] = \sum_n \sum_{k=0}^n kP(K = k)\ h_N(n)$

and $q_R(s) = \int_s^C (\int_{\frac{yL}{C}}^{\infty} g_X(x)\ dx)\ f_Y(y)\ dy$

Now, let us reformulate our initial equation.

$$O(s) = c_U\ E[U(s)] + c_R\ E[R(s)]$$

According to (8) :

$$O(s) = c_U\ E[U(s)] + c_R\ (q_R(s)\ E[D(s)])$$

Considering (9):

$$O(s) = c_U \; E[U(s)] + c_R \; (q_R(s) \; (E[K] - E[U(s)]))$$

Finally:

$$O(s) = c_U \; E[U(s)] + c_R \; q_R(s) \; E[K] - c_R \; q_R(s) \; E[U(s)]$$

$$O(s) = \underbrace{E[U(s)](c_U - c_R \; q_R(s))}_{\alpha} + \underbrace{c_R \; q_R(s) \; E[K]}_{\beta} \tag{11}$$

3 Calculation and Simulation

Our objective is to determine the threshold s that minimizes our criterion O(s). Function O(s) is too complex to be analyzed that's why simulation is used. In the next subsection, We restrict ourselves to a special case where the probability densities of the different random variables are uniformly distributed.

3.1 Special Case

Let us assume that the number of cars in a car station is uniformly distributed among the values $[n_m, n_M]$ where $n_m > 0$.

$$h_N(n) = \begin{cases} \frac{1}{n_M - n_m} & \forall \quad n \in [n_m, n_M] \\ 0 & \text{Otherwise} \end{cases}$$

This probability density is the consequence of a perfect balancing system. The distance of a trip varies from [0, G]. We assume that a car can cover distances which are uniformly distributed among 0 and **G** . Therefore, the probability density of the random variable **X** representing the distance that a client can cover is calculated as follows:

$$g_X(x) = \begin{cases} \frac{1}{G} & \forall \quad x \in [0, G] \\ 0 & \text{Otherwise} \end{cases} \tag{12}$$

If we replace $g_X(x)$ by its value in equation (5), we obtain :

$$
\begin{aligned}
P(X > \tfrac{yL}{C}/Y = y) &= \int_{\frac{yL}{C}}^{G} \tfrac{1}{G} dx \\
&= \begin{cases} 1 - \frac{yL}{GC} & s \leq y \leq \frac{GC}{L} \\ 0 & \text{Otherwise} \end{cases}
\end{aligned}
\tag{13}
$$

Remark:

The upper limit of the integral is **G** because, as we said before in (12), the probability density of the random variable X representing the distance that a client can cover has a strictly positive value when **x** lies between the distances 0 and G.

Assume that $f_Y(y)$, the probability density of the random variable **Y** representing the charge level of an available car, is determined as follows:

$$
f_Y(y) = \begin{cases} \frac{1}{C-s} & \forall \quad y \in [s, C] \\ 0 & \text{Otherwise} \end{cases}
\tag{14}
$$

Knowing that the charge level **y** needed to cover the maximal distance **G** is $\frac{CG}{L}$, we can reformulate equation (6) as follows:

$$
\begin{aligned}
q_R(s) = &\int_s^{\frac{CG}{L}} \left(P\left(X > \tfrac{yL}{C}/Y = y\right)\right) f_Y(y)\, dy \\
&+ \int_{\frac{CG}{L}}^{C} \left(P\left(X > \tfrac{yL}{C}/Y = y\right)\right) f_Y(y)\, dy
\end{aligned}
$$

But the second term is equal to zero because $P\ (X > \frac{yL}{C}/Y = y)$ is null when **y** is greater than $\frac{GC}{L}$ as demonstrated in (13). Finally we get:

$$q_R(s) = \int_s^{\frac{CG}{L}} \left(P\left(X > \frac{yL}{C} / Y = y\right)\right) f_Y(y)\, dy$$

$$q_R(s) = \int_s^{\frac{CG}{L}} \left(1 - \frac{yL}{GC}\right)\left(\frac{1}{C-s}\right) dy \tag{15}$$

$$q_R(s) = \begin{cases} \frac{(GC-sL)^2}{2LGC(C-s)} & \forall \quad y \in [s, \frac{CG}{L}] \\ 0 & \text{Otherwise} \end{cases}$$

3.2 Numerical Application

These are the values choosed for our initial simulation :

- Maximal distance of a trip **G** = 70 km.
- Maximal distance that a car can cover **L** = 100 km.
- Maximum level of charge **C** = 1.
- Unit cost when we have an unsatisfied client $c_U = 2$ pounds.
- Unit cost when a car runs out of energy during a trip $c_R = 5$ pounds.
- Maximal number of cars in a car station $n_M = 10$.
- Minimal number of cars in a car station $n_m = 1$.
- Maximal number of clients' demands in a car station **K** = 10.
- Probabilities of having **k** demands in a car station, starting from zero demands till 10, are [0.1, 0.2, 0.2, 0.1, 0.1, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05].

We will consider each term of equation (11) separately and see how the curves look like. For the illustration, we used scilab software.

Consider the first term α

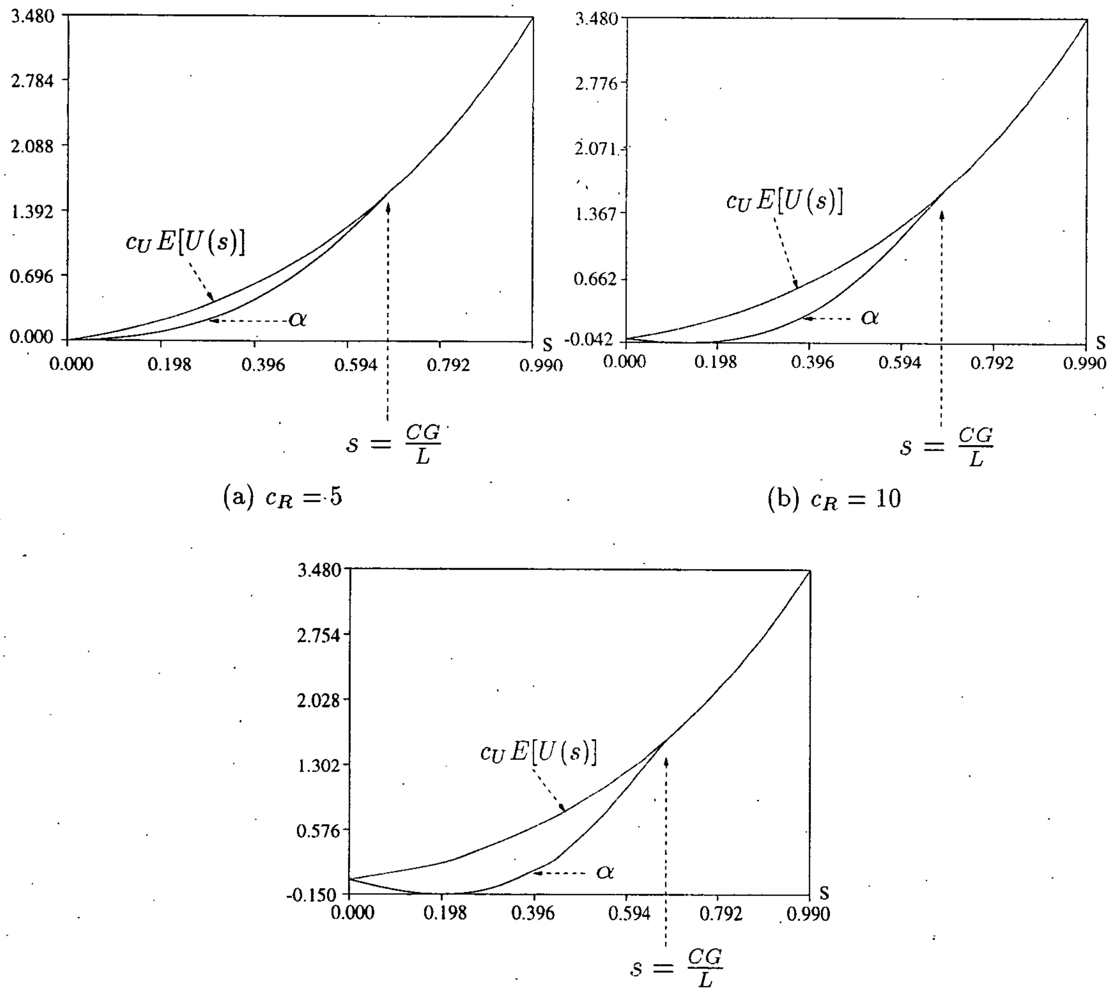
$$\alpha = E[U(s)](c_U - c_R\, q_R(s))$$

$$\alpha = c_U\, E[U(s)] - c_R\, q_R(s)\, E[U(s)]$$

where :

- E[U(s)] is the expectation of the number of unsatisfied clients in a car station due to the immobilization of some cars according to the threshold policy. When we raise the threshold **s**, the number of unsatisfied clients increases due to the increase in the number of unavailable cars.
- $q_R(s)$ is the probability that a car will run out of energy during a trip. This probability decreases as the threshold increases, and it becomes null when the threshold exceeds the charge level of $\frac{CG}{L}$. Remember that $\frac{CG}{L}$ is the charge level required to cover a trip of distance G.
- $c_U\ E[U(s)]$ is the cost when we have unsatisfied clients due to the unavailability of cars.
- $c_R\ q_R(s)\ E[U(s)]$ is the cost when unavailable cars - cars whose level of energy is less than the threshold - are allowed to leave the car station, and some of these cars run out of energy during the journey.
- The difference between the two previous terms, represents the gain obtained by preventing unavailable cars from leaving the car station and consequently from running out of energy during a journey.

In the following figures, we show how term α evolves at different thresholds, and how it reacts when varying the unit cost of a car breakdown during a trip. If the threshold is set at a value greater than $\frac{CG}{L}$, the two curves $c_U\ E[U(s)]$ and α are superposed because at this level the probability that a car will run out of energy during a trip is null. Also, we notice that the gain increases when the unit cost of a breakdown increases.



(a) $c_R = 5$

(b) $c_R = 10$

(c) $c_R = 15$

Figure 1: Term α

Now, consider the second term β

$$\beta = c_R \; q_R(s) \; E[K]$$

This term represents the cost when all clients' demands are satisfied due to the assumption that all the cars are available - their charge level is higher than the threshold set. Some of these clients will have a breakdown during the trip. The figure below shows the evolution of β at different thresholds.

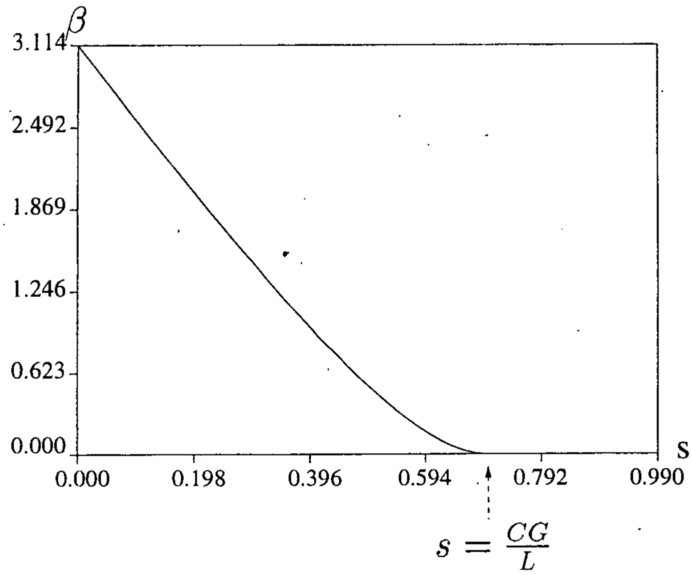


Figure 2: Term β of O(s)

It is obvious that when a car has a low charge level, its probability of running out of energy during a trip is high. The curve demonstrates that when the threshold is set low, the cost β is high and it decreases when the threshold increases. This cost β is null when the threshold is higher than $\frac{CG}{L}$ because the probability of running out of energy at this charge level is zero.

If we combine both terms (α and β), we will obtain figure (3). We could not prove the convexity of the curve although numerical results did it, so we will assume temporarily the convexity. By looking at the curve and noticing its smooth shape, we will use the dichotomy method to find the minimum point s_0. The first step of our procedure is to determine the interval of the threshold s which is initially [0,C], where C is the maximum level of energy. At the middle point $\frac{C}{2}$ we calculate the derivative of O(s). If the result is zero, this means that we have found the minimum point and we end the procedure. Otherwise, we check whether the slope at this point is decreasing or not ; if decreasing this point becomes the lower limit of the interval, otherwise it is the new upper limit.

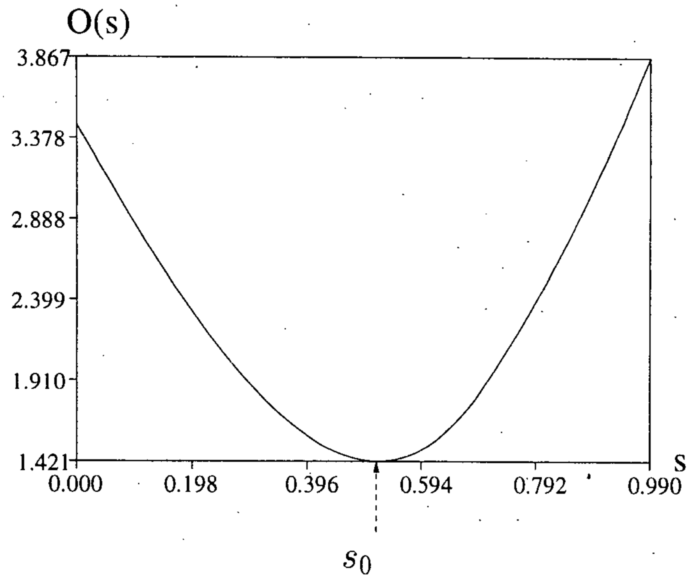


Figure 3: the O(s) function

The next simulation consists in determining the threshold **s** that minimizes the criterion O(s) at different values of $\frac{G}{L}$. We notice that when this ratio increases, in other words when **G** approaches **L** , the threshold increases.

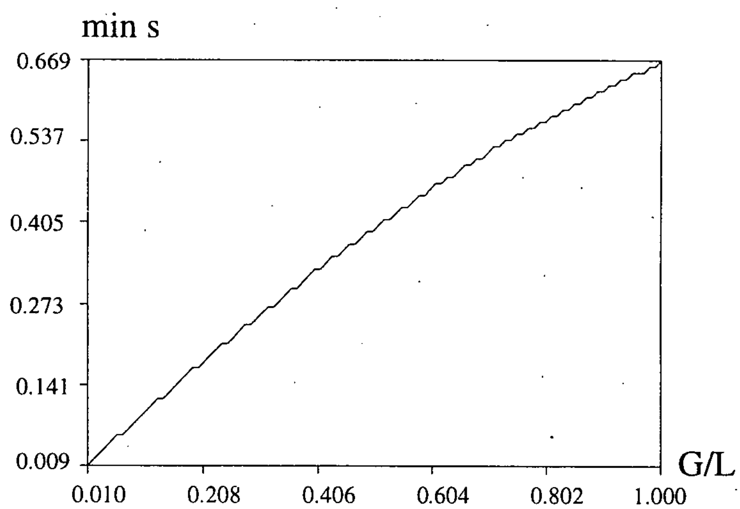


Figure 4: Relation between the ratio $\frac{G}{L}$ and the threshold s

The following simulation consists in determining the threshold s that minimizes the criterion O(s) at different values of $\frac{c_R}{c_U}$. Remember that c_R is the cost when a car runs out of energy during a trip, while c_U is the cost when a client does not find an available car at the car station. It is evident that c_R must be greater than c_U, because when a client has a breakdown during a trip, in addition to his dissatisfaction, the car must be towed as far as a station. We notice that when this ratio increases, in other words, when c_R is much greater than c_U, the threshold increases to minimize the breakdowns.

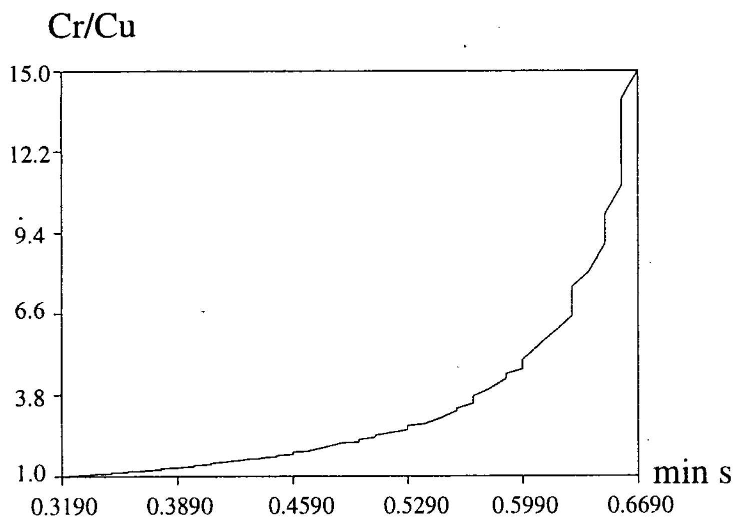


Figure 5: Relation between the ratio $\frac{c_R}{c_U}$ and the threshold s

3.3 General Case

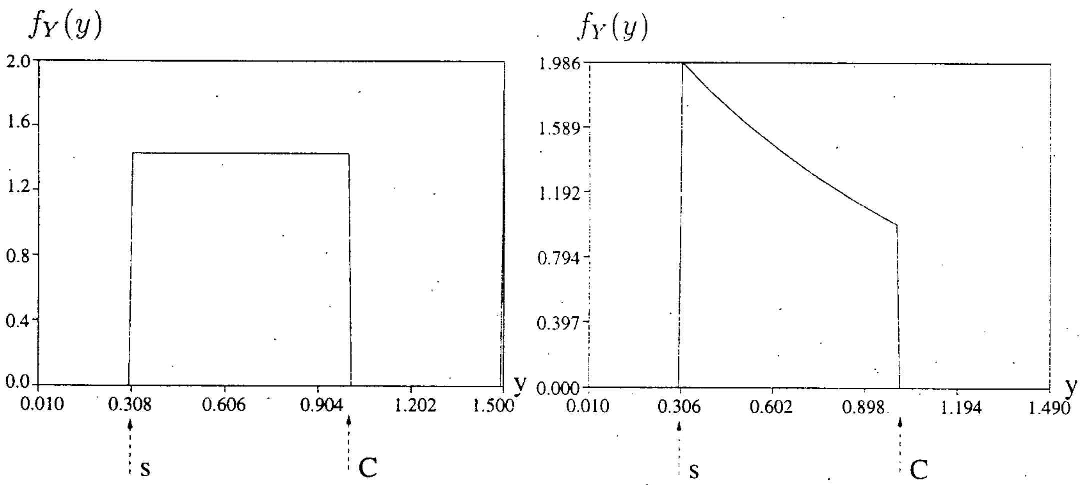
Now, we will vary the probability density $f_Y(y)$, (see 14) and see its effect on the criterion O(s) and the threshold.

The new probability density of the random variable **Y** representing the charge level of an available car

$$f_Y(y) = \begin{cases} \frac{e^{-y}}{(e^{-s}-e^{-C})} & \forall \quad y \in [s, C] \\ 0 & \text{Otherwise} \end{cases}$$

The two figures below compare the different $f_Y(y)$ for the random variable **Y** . The second one seems to be more realistic for the following reasons.

1. To reach the charge level **C**, a car needs a lot of time.
2. Suppose that we have a limited number of battery chargers. As soon as the charge level of a car reaches the threshold level, the process of charging stops. Therefore, an unavailable car can be charged.



(a) $f_Y(y) = uniform$

(b) $f_Y(y) = exponential$

Figure 6: Different $f_Y(y)$

For our previous calculations, only the calculation of $q_R(s)$, the probability that a car will run out of energy during a trip, will change.

$$q_R(s) = \int_s^{\frac{CG}{L}} \left(P\left(X > \tfrac{yL}{C} / Y = y\right)\right) f_Y(y)\, dy$$

$$q_R(s) = \int_s^{\frac{CG}{L}} \left(1 - \tfrac{yL}{GC}\right) \left(\tfrac{e^{-y}}{(e^{-s}-e^{-C})}\right) dy \tag{16}$$

$$q_R(s) = \begin{cases} \frac{GCe^{-s}+L(e^{\frac{-CG}{L}}-se^{-s}-e^{-s})}{GC(e^{-s}-e^{-C})} & \forall \quad y \in [s, \frac{CG}{L}] \\ 0 & \text{Otherwise} \end{cases}$$

The next figure shows the evolution of the criterion O(s) when we change $f_Y(y)$

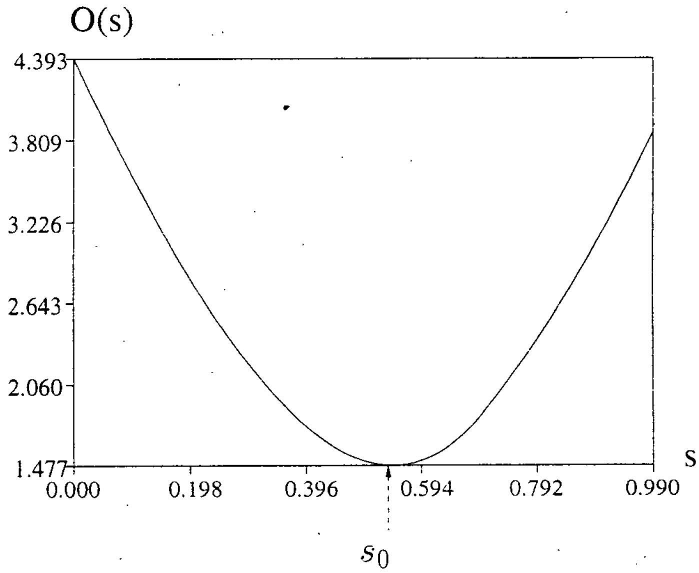


Figure 7: O(s) when $f_Y(y) = \frac{e^{-y}}{(e^{-s} - e^{-C})}$

In this figure we superpose both figures (3) and (7),

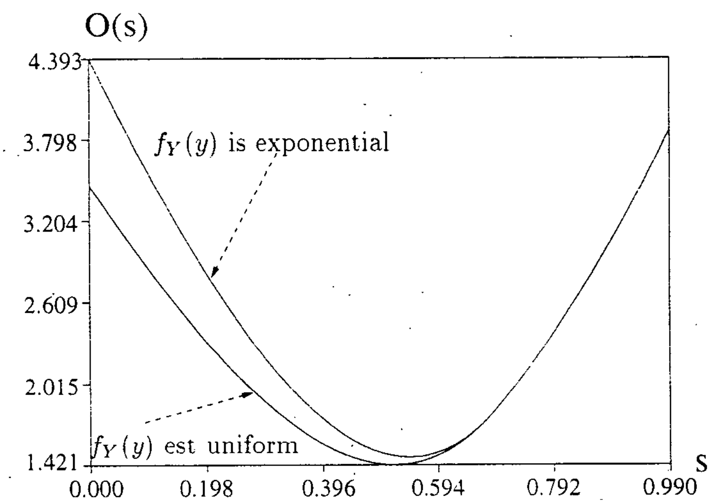


Figure 8: O(s) with two different $f_Y(y)$

Notice that the probability that a car will run out of energy is higher when using the exponential probability density. As a consequence, the values of the criterion O(s) and the minimal corresponding threshold, calculated when using the exponential probability density are higher than when calculated with the uniform distribution one. It is easy to understand the previous paragraph if we rewrite equation (11) as follows:

$$O(s) = c_U \; E[U(s)] + q_R(s) \; [c_R \; E[K] - c_R \; E[U(s)]]$$

4 Conclusion

We saw in this paper that determining a threshold is not an easy matter. To calculate this threshold, we defined a criterion based on different costs. Our goal was to find the threshold that minimized this criterion. We observed that we had to take into consideration two types of costs. The first one is when a client's demand is refused because the car present is not sufficiently charged, while the second one is when a client has a breakdown during a journey due to a shortage of energy. We noticed that these costs go in opposite directions. In other words, if we reduce a cost by choosing a certain threshold, we are increasing the other one. Consequently we have to make a compromise and find the threshold which minimizes the whole criterion.

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